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STATIONARY FLOW OF A REACTING LIQUID WHOSE PROPERTIES VARY WITH THE EXTENT OF REACTION

D. A. Vaganov

UDC 532.542:060.095.26

Considerable interest attaches to the flow of a reacting liquid whose properties change during the reaction in relation to analysis of displacement-type flow polymerization reactors. There is a substantial increase in viscosity as the polymerization proceeds (by up to a factor 10^6 or more), which produces qualitative changes in the flow picture, and this in turn influences the macrokinetic relationships.

Here we consider the simple case of isothermal flow of a reacting liquid in which the extent of reaction and the properties are uniquely determined by the reaction time. A general self-modeling solution is derived and the main features of the flow are examined for the case where there is a considerable increase in viscosity.

1. Consider the stationary laminar flow of a reacting Newtonian liquid in a tube (tubular flow reactor). The viscosity μ and density ρ alter from the initial values μ_0 and ρ_0 at the inlet to the final values μ_1 and ρ_1 on complete reaction. The temperature is taken as constant, and the reactions are independent of the velocity gradients, while the effects of diffusion are neglected because of the smallness of the diffusion coefficients. The extent of reaction and the properties of the liquid are uniquely determined by the reaction time t, and the relationships are considered as given.

To derive the flow pattern we assume that the radial velocity component arising from change in the flow profile on account of the change in properties is small by comparison with the axial component, while the pressure change along the radius is slight, and also that the viscosity is large enough for one to neglect inertia and the effects of the inlet hydrodynamic-stabilization part. With these assumptions, the flow at each section is essentially plane-parallel, which is an approximation widely used in various applications to the flow of liquids with varying properties [1-3]. The general equations of motion for a Newtonian liquid [4] in this approximation give

$$\frac{1}{R}\frac{\partial}{\partial R}\left(\mu R\frac{\partial V}{\partial R}\right)+\frac{dP}{dZ}=0,\quad 0\leqslant R\leqslant R_0,\quad 0<Z<Z_0, \tag{1.1}$$

where V is the axial component of the flow velocity, R is the distance from the axis, P = P(Z) is the difference between the pressure at the inlet and that in a given section, Z is the distance from the start, and Z_0 and R_0 are the tube length and radius correspondingly. The radial velocity component W is given by the equation of continuity

$$\frac{1}{R}\frac{\partial}{\partial R}(\rho RW)+\frac{\partial}{\partial Z}(\rho V)=0. \tag{1.2}$$

The reaction time t is the time from the instant when an element of the liquid enters the reactor and is given by

$$t=\int_0^Z\frac{dz}{V}, \tag{1.3}$$

in which the integration is carried out along the path of motion of that element, i.e., along a given flow line $\psi(Z, R)$ = const, where

Chernogolovka. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 96-103, January-February, 1984. Original article submitted October 21, 1982.

$$\psi(Z, R) = \int_R^{R_0} \rho V 2\pi r dr \tag{1.4}$$

is the streamline function, which is related to the velocity components by

$$\partial\psi/\partial R = -2\pi R\rho V, \quad \partial\psi/\partial Z = 2\pi R\rho W. \tag{1.5}$$

At the wall (at $R = R_0$), ψ is zero, while the value at the axis coincides with the mass flow rate $M = \pi R_0^2 \rho_0 U$, where U is the speed with which the liquid is supplied to the tube averaged over the cross section.

The spatial distribution of the reaction time t(Z, R) in accordance with (1.3) satisfies

$$V\frac{\partial t}{\partial Z} + W\frac{\partial t}{\partial R} \equiv V\left(\frac{\partial t}{\partial Z}\right)_\psi = 1, \tag{1.6}$$

and at the inlet (at Z = 0) we have the natural boundary condition

$$t(0, R) = 0 \quad \text{for all} \quad 0 \leqslant R < R_0. \tag{1.7}$$

Our task is to solve (1.6) together with (1.1) and (1.2) with given $\mu(t)$ and $\rho(t)$ relationships and then to analyze the general features of the flow. No constraints are imposed on the forms of $\mu(t)$ and $\rho(t)$; the particular case of a stepwise change in properties has been considered in [5].

2. The results of [5, 6] indicate that the solution to (1.6) should be sought as a self-modeling relationship

$$t = t(X), \quad X \equiv \psi(Z, R)/\sigma(Z) \tag{2.1}$$

such that

$$t \to 0 \quad \text{for} \quad X \to \infty, \; t \to \infty \quad \text{for} \quad X \to 0, \tag{2.2}$$

while the function $\sigma(Z)$ appearing in the self-modeling variable X, which is also to be found during the analysis, is a strictly monotonically increasing function and should satisfy the condition

$$\sigma = 0 \quad \text{at} \quad Z = 0, \tag{2.3}$$

in order that the value of X should increase without limit for $Z \to 0$ for all $\psi > 0$ and thus should meet the requirement of (1.7).

The equations for $\sigma(Z)$ and the self-modeling dependence t(X) are to be derived from (1.6), namely since in accordance with (2.1)

$$\left(\frac{\partial t}{\partial Z}\right)_\psi = -\frac{X}{\sigma(Z)}\frac{d\sigma(Z)}{dZ}\frac{dt(X)}{dX}, \tag{2.4}$$

and because the representation of (2.1) applies, the left side of (1.6) should split up into the product of two groups of cofactors, one of which is a function only of Z and the other of X. To obey (1.6) it is then necessary to specify a constant value for each of the groups of cofactors, which gives the desired equations.

According to (2.1) and (1.5), for constant Z

$$\frac{\partial V}{\partial R} \equiv \frac{\partial\psi}{\partial R}\frac{\partial V}{\partial\psi} = -2\pi R\rho V\frac{\partial V}{\partial\psi} = -\frac{\pi R\rho}{\sigma}\frac{\partial V^2}{\partial X}. \tag{2.5}$$

On the other hand, on integrating (1.1) with respect to R and using the fact that $\partial V/\partial R = 0$ for R = 0 we have

$$\partial V/\partial R = -(R/2\mu)dP/dZ. \tag{2.6}$$

Then on integrating the expression for $\partial V^2/\partial X$ implied by (2.5) and (2.6) we get

$$V^2 = \frac{\sigma}{2\pi\mu_0\rho_0}\frac{dP}{dZ}\int_0^X f(t(x))\,dx, \quad f(t) \equiv \frac{\mu_0\rho_0}{\mu\rho}. \tag{2.7}$$

This expression for V has the form necessary for the existence of the self-modeling solution of (2.1) and therefore on substituting (2.4) and (2.7) into (1.6) we get finally the following equations defining $\sigma(Z)$ and t(X):

$$\frac{1}{\sqrt{\sigma}} \frac{d\sigma}{dZ} \sqrt{\frac{dP/dZ}{2\pi\mu_0\rho_0}} = \beta; \tag{2.8}$$

$$-X \frac{dt}{dX} \sqrt{\int_0^X f(t(x))\, dx} = \frac{1}{\beta}, \tag{2.9}$$

where β is an arbitrary constant whose presence reflects the fact that conditions (2.2) and (2.3) allow σ(Z) to change by any constant positive factor, since the form of (2.1) is then unaltered. However, if the value of β is specified, (2.8) together with (2.3) uniquely determines σ(Z), and therefore the value of X, and for definiteness in what follows we put β = 4. In that case, in spite of the differences in the physical formulation and the form of the self-modeling variable, (2.9) differs from the self-modeling equation arising in [6] only in the behavior of f(t).

We differentiate (2.9) and eliminate the value of the integral to get an ordinary differential equation, which can be put in the following form with time t taken as independent variable:

$$y \frac{d^2 y}{dt^2} = \left(\frac{dy}{\partial t}\right)^2 - f(t), \quad y \equiv \frac{1}{2\sqrt{X}}, \tag{2.10}$$

where the new variable y is completely equivalent to X but has a mode of variation more convenient for analysis.

The solution to (2.10) should be monotone and satisfy the conditions

$$y \to 0 \text{ for } t \to 0, \quad dy/dt \to \sqrt{f(\infty)} \text{ for } t \to \infty. \tag{2.11}$$

The first of these boundary conditions corresponds to (2.2), while the second follows from (2.9) in the limit X → 0. Also, for y → 0 it follows from (2.10) that

$$\frac{dy}{dt} \to \sqrt{f}, \quad \frac{d^2 y}{dt^2} \Big/ \frac{df}{dt} \to \frac{1}{\sqrt{f}} \tag{2.12}$$

and also that for f(t) ≡ const the general solution to (2.10) is as follows (C and φ are arbitrary real constants):

$$C^{-1}\sqrt{f} \sin(Ct + \varphi), \quad C^{-1}\sqrt{f}\, \mathrm{sh}\,(Ct + \varphi), \text{ and } \sqrt{f}\, t + \varphi. \tag{2.13}$$

An infinite set of integral curves arises from the point y = t = 0, which is a singular point for (2.10). Some of these reach the y = 0 axis again, and it can be shown that the envelope $y_e(t)$ of the family of such nonmonotone integral curves is a unique solution to this boundary-value problem.

To prove this we note that if $y_1(t)$ and $y_2(t)$ are integral curves for (2.10) with in general different functions $f_1(t)$ and $f_2(t)$ and these curves at some $t = t_0$ touch one another, then because (2.10) can be put in the form

$$d^2 \ln y/dt^2 = -y^{-2} f, \tag{2.14}$$

and we have for the ratio $\eta = y_1(t)/y_2(t)$ that

$$\frac{d}{dt} \ln \eta(t) \equiv \frac{y_1'(t)}{y_1(t)} - \frac{y_2'(t)}{y_2(t)} = \int_{t_0}^{t} \{(\eta^2 - 1) f_2 + (f_2 - f_1)\}\, y_1^{-2} d\tau, \quad \eta(t_0) = 1, \tag{2.15}$$

it follows that if $f_2 \geqslant f_1$ then for $t > t_0$ we have $y_1 \geqslant y_2$ and $y_1' \geqslant y_2'$, and equality occurs only if there is complete coincidence between f_1 and f_2 on the part between t_0 and t. On this basis, we use the corresponding solutions of (2.13) to get that if the first of the following inequalities is violated

$$\min_{t > t_0} \sqrt{f(t)} \leqslant y'(t_0) \leqslant \max_{t > t_0} \sqrt{f(t)} \tag{2.16}$$

the integral curve for $t > t_0$ is majored from above by a certain sinusoidal function, while when the second condition is violated it increases exponentially, and consequently in both cases it cannot be either the envelope y_e or the solution to the boundary-value problem.

Then the envelope $y_e(t)$ should satisfy (2.16) for all values of the argument, which means that it satisfies the boundary conditions of (2.11), i.e., it is the desired solution. If on the other hand we assume that two integral curves for (2.10) y_1 and y_2 satisfy (2.11), then the ratio $\eta = y_1(t)/y_2(t)$ is given by (2.14) and (2.11) as an expression coincident with (2.15) for $t_0 = \infty$ and $f_2 \equiv f_1$, and from this for $\eta \neq 1$ and $f > 0$ it follows that $d|\ln \eta(t)|/dt < 0$. Consequently, for $y_1 \not\equiv y_2$ we should inevitably have $\eta(0) \neq 1$ for these integral curves. However, in fact we have $y_1/y_2 \to y_1'(0)/y_2'(0) = 1$ for $t \to 0$ from (2.11) and (2.12), which is a contradiction serving as proof that the solution is unique.

The existence and uniqueness of a solution to the boundary-value problem are simultaneously a proof of existence for the self-modeling dependence of (2.1).

3. According to this self-modeling solution, the reaction-time distribution in the flow of liquid through any section Z = const is defined by the same function t(X); as Z increases, we merely have to discard a larger and larger part of the universal curve, while the relative values of the density distribution remain unchanged. The mean time required to reach a given section is

$$\vartheta(Z) \equiv M^{-1}\int_0^M t\,d\psi = X_0^{-1}\int_0^{X_0} t(X)\,dX, \quad X_0 \equiv M/\sigma(Z). \tag{3.1}$$

Other characteristics in addition to ϑ can be represented in quadratures as functions of X and X_0 (or $y_0 \equiv 1/2\sqrt{X_0}$). For example, since in accordance with (2.1) and (1.5) for constant Z we have

$$\partial R^2/\partial X = \sigma\partial R^2/\partial\psi = -\sigma/\pi\rho V, \tag{3.2}$$

then on substituting the above expression (2.7) into (3.2) and integrating we have

$$(R_0^2 - R^2)\sqrt{\frac{dP}{dZ}} = \sqrt{\frac{2}{\pi}\frac{\mu_0}{\rho_0}}\,\sigma G(X), \tag{3.3}$$

where

$$G(X) = \int_0^X \frac{\rho_0}{\rho(t(x))}\frac{dx}{\sqrt{F(x)}}, \quad F(X) = \int_0^X f(t(x))\,dx. \tag{3.4}$$

We should have R = 0 for $X = X_0$, and therefore (3.3) gives the pressure gradient dP/dZ as

$$\frac{dP}{dZ} = \frac{2\mu_0 M}{\pi R_0^4\rho_0}\frac{G^2(X_0)}{X_0} = 8\frac{\mu_0}{R_0^2}U\Phi^2, \quad \Phi \equiv y_0 G(X_0), \tag{3.5}$$

while for the radial coordinate we get

$$R^2/R_0^2 = 1 - G(X)/G(X_0). \tag{3.6}$$

Substitution of (3.6) into (2.7) gives the flow speed as

$$V/U = \sqrt{F(X)}G(X_0)/X_0. \tag{3.7}$$

From (2.8) with (3.5) we have for the Z coordinate that

$$Z = U\zeta(y_0), \quad \zeta(y_0) = 2\int_0^{y_0}\Phi(y)\,dy, \tag{3.8}$$

while from (3.5) and (3.8) we have for the pressure change that

$$P = 8\frac{\mu_0}{R_0^2}U^2 p(y_0), \quad p(y_0) = 2\int_0^{y_0}\Phi^3(y)\,dy. \tag{3.9}$$

For a given flow rate, (3.5)-(3.9) completely describe the variations in the characteristics along the tube. Finally, the relation between the flow rate and the pressure difference $P_0 \equiv P(Z_0)$ is given by a parametric representation following from (3.8) and (3.9):

$$U=\frac{Z_0}{\zeta(y)},\quad P_0=8\frac{\mu_0}{R_0^2}Z_0^2\varkappa(y),\quad \varkappa\equiv\frac{p}{\zeta^2}\quad (0<y<\infty). \tag{3.10}$$

Therefore, all the main flow parameters are expressed in quadratures in terms of the above t(X) relation.

4. Particular interest attaches to the regularities when $\mu_1 \gg \mu_0$; the flow of unreacted liquid in that case is compressed into a narrow jet breaking through the largely immobile layer of products, and in order to attain complete transformation one has to specify that the average residence time is substantially larger than the time actually necessary for reaction [5].

We assume that after a certain time t_0 the properties cease to alter (complete reaction), and for simplicity we assume that the density is constant, so we examine the asymptotic behavior for $A \equiv \mu_1/\mu_0 \to \infty$ on the assumption that the viscosity is unchanged at times for which the function $f(t) = \mu_0/\mu(t)$ appreciably exceeds $\alpha \equiv A^{-1} \ll 1$ [for example, by considering the sequence of functions $f(t, \alpha) = \max\{\alpha, f_0(t)\}$, where $f_0(t)$ is independent of α and becomes zero for $t > t_0$].

In accordance with (2.13), for $t > t_0$

$$y=y_*+\sqrt{\alpha}(t-t_0), \tag{4.1}$$

where $y_* \equiv y(t_0)$ and since the behavior of the solution over the final part $t \leqslant t_0$ for sufficiently small α virtually ceases to be dependent on α with the above assumptions, y_* can be considered as approximately constant. In the case of a stepwise change in viscosity [5] $y = t_0/\arccos\alpha \simeq (2/\pi)t_0$, and by means of analogy we assume also that $y_* \equiv (2/\pi)t_*$ in the general case. The values of t_* for a series of relationships have been given in [6]; in particular, for the linear relationship $f(t) = 1 - t/t_0$ we have $t_* \simeq 0.60t_0$.

The value of $\zeta \equiv Z/U$ for a constant density coincides with the mean time required to attain a given section $\zeta\equiv\vartheta$, and we have as follows from (3.1) with (4.1) for the distance Z_* at which complete reaction is attained:

$$\zeta_*=t_0+y_*\sqrt{A}\simeq(2/\pi)t_*\sqrt{A}. \tag{4.2}$$

For $Z > Z_*$ we have the usual Poiseuille flow of completely reacted liquid, so

$$p=p_*+A(\zeta-\zeta_*),\quad p_*\equiv p(\zeta_*), \tag{4.3}$$

and for $X > X_* \equiv (\pi/4t_*)^2$ we represent the expression corresponding to (3.1) for $\zeta < \zeta_*$ in the form

$$\zeta=\zeta_0(X)+\frac{X_*}{X}\zeta_*,\quad \zeta_0(X)=X^{-1}\int_{X_*}^{X}t(x)\,dx, \tag{4.4}$$

and note that apart from a narrow range $X \simeq X_*$, in which the first term in (4.4) for $\alpha \ll 1$ can be neglected because of its smallness, $\zeta_0(X)$ is virtually independent of α. Similarly, (3.4) for $X > X_*$ can also be represented in the following form apart from quantities that tend to zero for $\alpha \to 0$:

$$G(X)=G_0(X)+G_*,\ G_*=(\pi/2)\sqrt{A}/t_*,\ F(X)\simeq F_0(X), \tag{4.5}$$

where $G_0(X)$ and $F_0(X)$ are independent of α. For X sufficiently large

$$\zeta_0(X)\simeq 1/\sqrt{X},\quad G_0(X)\simeq 2\sqrt{X}, \tag{4.6}$$

where the substitution of (4.6) into (4.4) and (4.5) leads to uniformly suitable asymptotic expansions [the terms in (4.4) and (4.5) are comparable only for $X/X_* \sim A \gg 1$, and therefore for values of the argument at which differences from (4.6) appear the values of $\zeta_0(X)$ and $G_0(X)$ for $A \to \infty$ are negligibly small by comparison with the second terms]. As a result, we get from (3.9) for $Z < Z_*$ from (4.4)-(4.6) and (4.2) that

$$p=\zeta+\zeta^2(\pi/4t_*)\sqrt{A}. \tag{4.7}$$

As regards the motion, the distance Z_* for $A \gg 1$ can be subdivided into two partially intersecting regions corresponding, respectively, to the values $X_0/X_* \gg 1$ where the approximation of (4.6) applies and values $1 \leqslant X_0/X_* \ll A$ for which one can neglect the first terms in (4.4) and (4.5). In the first region, i.e., in the initial part $0 < \zeta \ll \zeta_*$, the flow of the reacting liquid differs little from that of an inert flow in a channel of variable cross section:

$$\frac{dP}{dZ} \simeq 8\frac{\mu_0}{R_*^2}\frac{M}{\pi R_*^2 \rho}, \quad \frac{R_*^2}{R_0^2} \equiv \frac{G_0(X_0)}{G(X_0)} \simeq \left(1 + \frac{\pi}{2}\frac{Z}{Ut_*}\sqrt{A}\right)^{-1/2}, \tag{4.8}$$

while in the second, i.e., in the part $t_*/\sqrt{A} \ll \zeta \leqslant \zeta_*$, which constitutes much the larger part of Z_*, almost all of the cross section is occupied by completely reacted liquid ($R_*^2/R_0^2 \ll 1$), and the mass flow rate of this $M_* \equiv \psi(Z, R_*)$ defines the current value of the pressure gradient:

$$\frac{dP}{dZ} \simeq 8\frac{\mu_1}{R_0^2}\frac{M_*}{\pi R_0^2 \rho}, \quad \frac{M_*}{M} \equiv \frac{X_*}{X_0} \simeq \frac{Z}{Z_*}.$$

Under otherwise equal conditions, the compression and acceleration of the flow of a liquid which has been converted to a narrow jet are proportional to $\sqrt{A}$:

$$\frac{R^2}{R_0^2} \simeq \frac{G_0(X_0) - G_0(X)}{(\pi/2t_*)\sqrt{A}}, \quad \frac{V}{U} \simeq \frac{\sqrt{F_0(X)}}{X_0}\frac{\pi\sqrt{A}}{2t_*},$$

and although the detailed behavior of the jet is dependent on the detailed form of f(t) [for example, for a stepwise change the maximum value of $v = V/U\sqrt{A}$ is one and is attained at $\zeta = (1/2)\zeta_*$, while for a linear relationship it is 0.7 and is attained at $\zeta \simeq 0.4\zeta_*$], while the properties such as constancy of the effective reaction rate dM_*/dZ along the jet are retained.

The following are the maximum values of the flow rate and pressure difference for which there is still complete conversion at the exit, in accordance with (3.10):

$$U_* \simeq \frac{\pi}{2t_*}\frac{Z_0}{\sqrt{A}}, \quad P_* \simeq 8\frac{\mu_0}{R_0^2}Z_0^2\frac{\pi\sqrt{A}}{4t_*} = 8\frac{\mu_1/2}{R_0^2}Z_0 U_*,$$

while we have as follows for the dependence of the pressure difference on the flow rate from (4.7) and (4.3) for $U > U_*$ and $U < U_*$ correspondingly:

$$\frac{P}{P_*} = 1 + \frac{2}{A}\left(\frac{U}{U_*} - 1\right) \text{and} \frac{P}{P_*} = 2\frac{U}{U_*} - \left(\frac{U}{U_*}\right)^2. \tag{4.9}$$

It follows from (4.9) that for $A \gg 1$ there is a wide range in $1 \leqslant U/U_* \ll A$ corresponding virtually to the same pressure difference $P \simeq P_*$, so if the flow is maintained by specifying the pressure difference, on passing through this value the flow rate increases (or decreases) sharply by about a factor A, while the mode of flow changes stepwise from that corresponding to complete conversion at the output to one in which the only visible sign of reaction is the presence of an almost immobile layer of products compressing the flow [$Z_0/U \ll \zeta_*$, profile of the layer given in (4.8)].

One predicts a relationship of the form $\eta = aU_*/U$ for the extent of reaction η, where the factor a varies slightly with the flow rate, $a(U_*) = 1$.

The results imply that the major quantitative relationships in the case of a considerable increase in viscosity are determined by the values of only two parameters: the relative increase in viscosity $A \equiv \mu_1/\mu_0$ and the parameter t_*, which acts as a characteristic time scale. As t_* is the sole parameter reflecting the variation of viscosity in time, an important point is that its value is determined mainly by the behavior of the liquid at times corresponding to small extents of reaction, when the viscosity is still comparatively close to the initial value (also, the sooner the viscosity begins to differ appreciably from the initial value, the smaller t_* [6]), while there is hardly any effect on t_* from features of the viscosity change at high degrees of reaction, where the viscosity substantially exceeds the initial value, and the same applies to the time of complete reaction, which also has little effect on this parameter and consequently on the flow laws.

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FORCE CHARACTERISTICS AND FLOW PARAMETERS IN COMBUSTION MODELS

V. K. Baev, V. V. Shumskii,
and M. I. Yaroslavtsev

UDC 536.46:621.45.022

In [1] we find results on the force characteristics and flow parameters in the internal section of a gasdynamic model with combustion in a pulsed wind tunnel at a Mach number in the incident flow M_i = 7.3, the combustion being determined because the energy aspects of the IT-301 pulsed tunnel [2] allow one to have a four-chamber volume of not more than about 1.2 dm^3 subject to the condition of providing pressures and temperatures sufficient for self-ignition of the hydrogen in the model. This four-chamber volume provides a diameter for the critical nozzle section of not more than about 10 mm on the basis of the maximum possible rate of fall in the incident air flow parameters allowing the forces and pressures to be measured, which with the model dimensions of [1] d_0 = 72 mm gave the minimum possible value at M_i = 7-7.5, where d_0 is the diameter of the inlet to the air intake in the model.

The positive results of [1] as regards the internal working process and force characteristics lead one to ask whether one can reduce M_i by reducing the dimensions of the model with the same tunnel energy and with the same rates of fall in the parameters? Therefore, a model with d_0 = 23 mm was devised, which enabled one to reduce the Mach number of the incident air flow to M_i = 4.9. The present paper is based on the results obtained with this model in a high-enthalpy air flow.

The model (Fig. 1) is a combination of the air intake and a chamber placed between Secs. 2 and 5, in which the hydrogen supplied to the model burns. The inner part of the model either had the critical section at the exit (form 1) or did not have a critical section (form 2); then the expansion factors in the combustion chamber $\overline{F}_c = F_5/F_2$ were 2.92 and 3.75 for forms 1 and 2 correspondingly, where F_5 and F_2 are the areas of cross section at the exit from the combustion chamber in Sec. 5 and at the inlet in Sec. 2.

The model was set up in the working part of the pulsed tunnel along the axis of the profiled nozzle on the lateral pillar 1 covered by the aerofoil 2. The pillar was attached to a mechanical device serving to isolate the longitudinal component from the total aerodynamic force. This device in turn was connected to a one-component aerodynamic balance [3]. The model contained no volume where the hydrogen could be stored, in contrast to the model of [1], so the hydrogen was supplied from an external cylinder of capacity 120-150 cm^3 through the supporting pillar.

The hydrogen was injected through two supply rings 3 and 4 through holes of diameter 1 mm drilled at 45° in the opposite direction to the flow: In the front ring 3 there were eight holes, and in the rear ring 4 there were six. About 60% of the hydrogen was supplied through the front ring and about 40% through the rear one.

The experiments were performed with the following ranges in the incident air parameters: stagnation pressure and temperature $p_{0i}(\tau)$ = 60-70 MPa, $T_{0i}(\tau)$ = 1850-1000°K, static temperature and pressure $p_i(\tau)$ = 1100-150 hPa, $T_i(\tau)$ = 350-180°K, dynamic head $q_i(\tau)$ = 1.8-0.25 MPa, M_i = 4.9, air flow rate through model 0.85-0.14 kg/sec, and Reynolds number $Re(\tau) = v_i(\tau)/\nu_i(\tau) = (100-30)\cdot 10^6$ 1/m, where v_i and ν_i are the velocity and kinematic viscosity of the incident air. Here τ = 0-50 msec is the current time, the origin being reckoned from the instant of discharging a capacitor bank in the forechamber. The rates of fall in the parameters

Novosibirsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 103-109, January-February, 1984. Original article submitted December 22, 1982.

